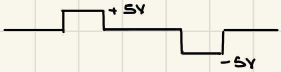
TRASMISSIONI

Problema delle telecomunicazioni: trasmettere bit

SEGNALI FISICI $\longleftrightarrow$ LIVELLI LOGICI



"1" "0"

Obbiettivo: ridurre il numero di bit

↓

CODIFICA DI SORGENTE

Jpeg

perdita di informazione

② LOSSY

zip

senza perdita di informazione

① LOSSLESS + importanti

① misura informazione → ENTROPIA h [bit]

codifica informazione $n = 1, 2, \dots, N$ (SI, NO)

$h = \log_2 N$ cioè bit necessari a codificare l'informazione

$$\left[h_n = \log_2 p_n \right]$$ inform. del singolo "messaggio" → evento

$$\left[H = E[h_n] \right]$$ {bit}

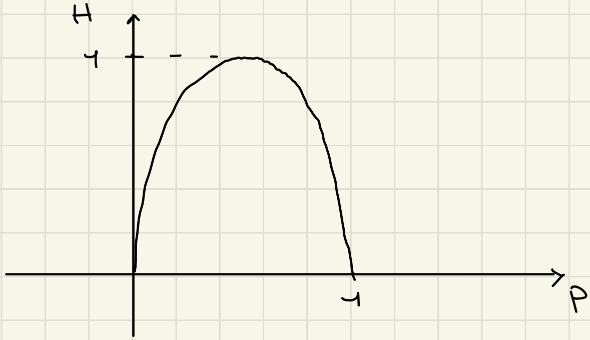
1011011110 sorgente binaria

singolo carattere "0" (p), "1" $(1-p)$

$$\left.\begin{array}{ll} \text{"0"} & (p) \\ \text{"1"} & (1-p) \end{array}\right\} \quad H = P_0(-\log_2 P_0) + P_1(-\log_2 P_1)$$

1 carattere sorgente binaria

$$\left[H = -p \log_2 p - (1-p)\log_2(1-p) \right]$$



2 caratteri

$$h_{x,y} = -\log_2 P(x,y)$$

$$= -\log_2\left(P(x|y) \cdot P(y) \right)$$

se indip

$$= -\log_2 P(x) - \log_2 P(y)$$

$$\left[H_{x,y} = -\sum_i \sum_J \left(P(x_i, y_J) \log P(x_i, y_J) \right) \right]$$

$$H_x H_y \leq H_{x,y} \leq H_x + H_y$$

$$\left[\bar{H} = \lim_{N \to +\infty} \frac{H(x_{-N+1}, x_{-N+2}, \dots, x_N)}{2N+1} \right]$$

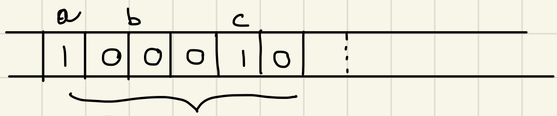
<u>SHANNON</u> $\longrightarrow$ codifica sorgente

esiste un codificatore ottimo

ridondanza media $\geq$ entropia

$$\left[R = \sum_{i=1}^{N+1} l(n_i)\, P(n_i)\right] \geq H$$

l ↓ lunghezza messaggio



a $\boxed{1}$ → n $\qquad l(n_1) = 1$

b $\boxed{0|0}$ $\qquad l(n_2) = 2$

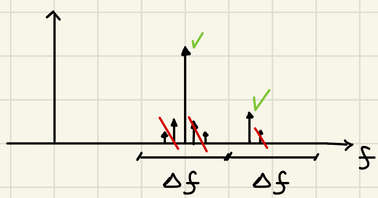
c $\boxed{0|1|0}$ $\qquad l(n_3) = 3$

(a, b, c)

↕ **codifica Huffman**

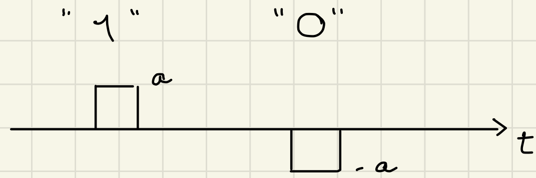
(1, 00, 010)

② **codifica Mp3** (audio)



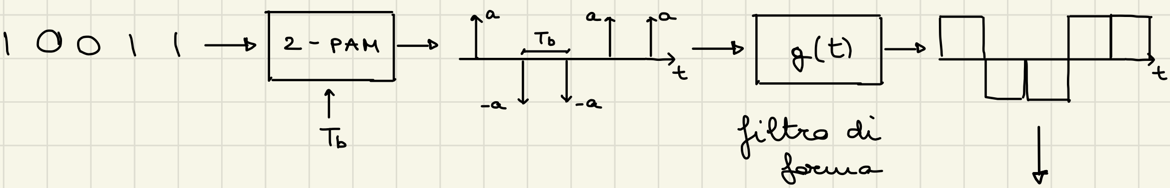
tengo solo la frequenza più importante in ciascun intervallo Δf

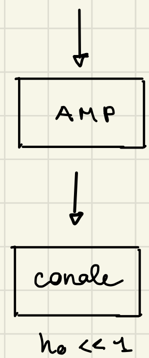
TRASMISSIONE NUMERICA



- Velocità di trasmissione R_b (bitrate bit/s)
- Probabilità di errore

Trasmissione banda base (ideale)

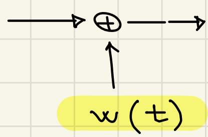




2-PAM: Pulse o Amplitude Modulation

$T_b = \frac{1}{R_b}$ tempo di bit

RUMORE



w(t) rumore termico elettronico

A W G Noise

additive

white

gaussian

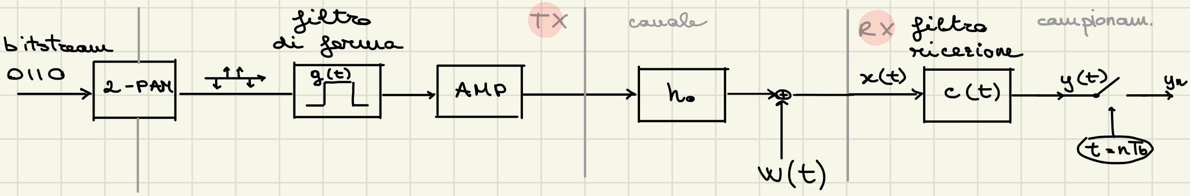
$w(t) \sim \mathcal{N}(0, \sigma_w^2)$

$$\sigma_w^2 = \frac{N_0}{2}$$

$$E[x] = \lim_{N \to \infty} \frac{1}{N} \sum_i^N x_i$$

$$w(t) \longrightarrow \boxed{h(t)} \longrightarrow w_h(t) = \int_{-\infty}^{+\infty} w(\tau)\, h(t-\tau)\, d\tau$$

$$E[w_h(t)] = \emptyset \qquad \sigma_{w_h}^2 = \sigma_w^2 \overbrace{\int (h(\tau))^2 d\tau}^{E_h}$$



c(t) deve massimizzare $\dfrac{\{[\pm a h_0 g(t) + w(t)] * c(t)\}^2 \big|_{t=\bar{t}}}{\sigma_{w_c}^2}$

che è il rapporto segnale rumore (SNR)

$$SNR = \frac{a^2 h_0^2 \{g(t) * c(t)\}^2 \big|_{t=\bar{t}}}{\frac{N_0}{2} E_c} \qquad \sigma_{w_c}^2 = \frac{N_0}{2} E_c$$

$$\leq \frac{a^2 h_0^2}{\frac{N_0}{2}} \frac{\int |g(t)|^2 dt \cancel{\int |c(t)|^2 dt}}{\cancel{\int |c(t)|^2 dt}} = \frac{a^2 h_0^2}{\frac{N_0}{2}} E_g \longrightarrow \left[c(t)_{\text{ottimo}} = g(-t) \right]$$

$$x(t) = \left(\sum_{n=-\infty}^{+\infty} b_n\, g(t - nT_b) \cdot h_0 \right) + w(t)$$

$$y(t) = y_s(t) + y_w(t) = x(t) * c(t) = x(t) * g(-t)$$

autocorrelazione di g — se g è un rect l'autocorrelaz. è un tri

$$y_s(t) = \left(\sum_{n=-\infty}^{+\infty} b_n\, g(t - nT_b) \cdot h_0 \right) * g(-t) = \boxed{\sum_{-\infty}^{+\infty} b_n h_0\, r_g(t - nT_b)}$$

massimo in 0 → $t = nT_b$

⇓

y_n

$$y_w(t) = w(t) * g(-t) \sim \mathcal{N}\left(0, \frac{N_0}{2} E_g = \sigma_{y_w}^2\right)$$

autocorrelazione in 0

$$y_n = b_n h_0 \underset{\uparrow}{E_g} + y_w \qquad y_w \to \mathcal{N}(0, \sigma^2_{y_w})$$

y_n DATO devo distinguere "1" e "0"

$$P("1"|y_n) \underset{"0"}{\overset{"1"}{\gtrless}} P("0"|y_n)$$

TX livello 1

$$P(y_n|_{"1"}) \frac{\cancel{P("1")}}{\cancel{P(y_n)}} \gtrless P(y_n|_{"0"}) \cdot \frac{\cancel{P("0")}}{\cancel{P(y_n)}}$$

"1" e "0" equiprobabili

$$\boxed{P_\varepsilon = Q\left(\frac{a h_0 E_g}{\sqrt{E_g \frac{N_0}{2}}}\right) = Q\left(\sqrt{\frac{a^2 h_0^2 E_g}{N_0/2}}\right)}$$

probabilità di errore

$$Q\left(\frac{m}{\sigma}\right)$$

Es: AUDIO CD 16 bit/sample 2 canali stereo

44000 sample/s $\qquad \varepsilon = 1$ bit/h

$h_0 = -90$ dB $\qquad N_0/2 = -200$ dB [W] $\qquad P_T = ?$

$$R_b = \underset{\text{sample/s}}{44000} \cdot \underset{\text{bit/sample}}{16} \cdot \underset{\text{canali}}{2} = 1,4 \text{ Mbit/s}$$

$$P_E = \frac{1\ bit/h}{1{,}4\cdot 10^6\ bit/s \cdot 3600\ s/h} = 2\cdot 10^{-10}$$

$$P_E = Q\left(\sqrt{\frac{a^2 h_0^2 E_g}{N_0/2}}\right) = 2\cdot 10^{-10}$$

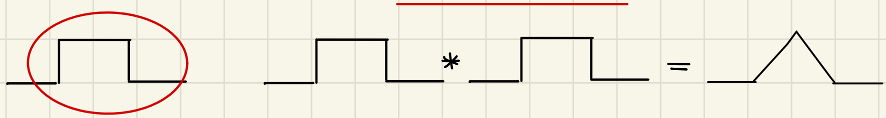
↳ devo trovare l'argomento (su Matlab: qfuncinv$(2\cdot 10^{-10})$)

$$\left.\begin{aligned} \frac{a^2 h_0^2 E_g}{N_0/2} &= (5{,}7)^2 \\ P_T = \frac{a^2 E_g}{T_b} &= a^2 E_g R_b \end{aligned}\right\} \Rightarrow \frac{P_T/R_b \cdot h_0^2}{N_0/2} = (5{,}7)^2$$

$$P_T = \frac{R_b}{h_0^2}\,\frac{N_0}{2}\cdot 5{,}7^2$$

$$P_T = 0{,}5\,mW$$

Tx banda base mezzo ideale h_0



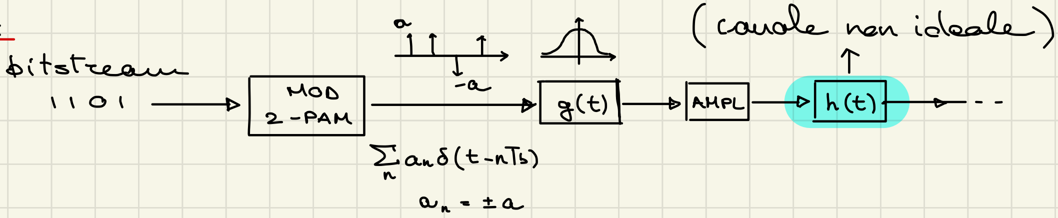
- Tx banda base mezzo non ideale (passa basso) $h(t)$

- Tx multilivello

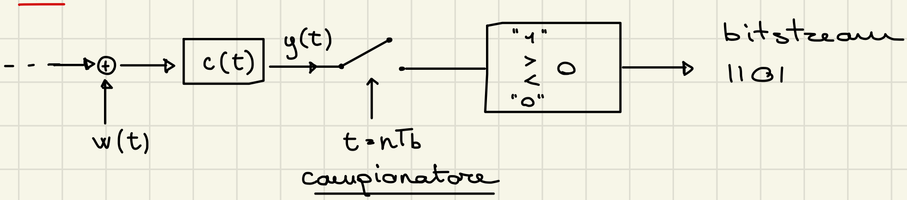
- Tx banda traslata

Trasmissione banda base (non ideale)

Tx



Rx



$$y(t) = \underbrace{\left(\sum_n a_n \delta(t-nT_b)\right) * \boxed{g(t) * h(t) * c(t)}^{\,d(t)}}_{\substack{\text{SEGNALE} \\ y_s(t)}} + \underbrace{w(t) * c(t)}_{\substack{\text{RUMORE} \\ y_n(t)}}$$

$$y_s(t) = \sum_m a_m\, d(t - mT_b)$$

cambio di pedice per non confondere con il tempo di campionamento $t = nT_b$

$$y_s(nT_b) = \sum_m a_m\, d(nT_b - mT_b)$$

$n = 0$ (1° bit) $\longrightarrow$ $y_s = \sum_m a_m d(-mT_b) = a_0 d(0) + \sum_{m\neq 0} a_m d(-mT_b)$

0

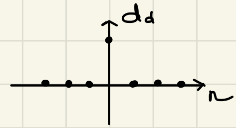
interferenza intersimbolica

per non avere interferenza intersimbolica (ISI)

$\Longrightarrow$ NO ISI cond. suff. $\boxed{\begin{aligned} d(mT_b) &= 1 \quad m = 0 \\ &= 0 \quad m \neq 0 \end{aligned}}$

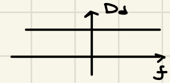
Soluzioni possibili: $d(t) = \mathrm{sinc}\left(\frac{t}{T_b}\right)$ (meno affidabile), $d(t) = \mathrm{sinc}^2\left(\frac{t}{T_b}\right)$ (più affidabile) digitale $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ analogico

$d_d(n)$ campionamento mT_b di $d_a(t)$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\shortparallel$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $d(t)$



$$d_a(t)\Big|_{t=nT_b} = d_d(n) = \delta(n) \longrightarrow \mathcal{F}[\delta(n)] = \text{const.}$$

D_d

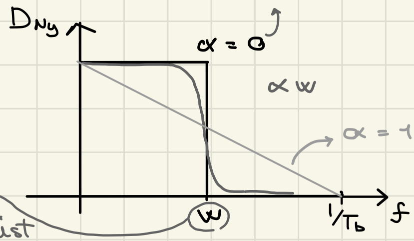


$\hookrightarrow$ la ripetizione nelle frequenze di D_a deve essere costante

$\hookrightarrow$ $\boxed{\sum_k D_a\left(f - \frac{k}{T_b}\right) = \text{const.}}$ Criterio di Nyquist

trasformata di $d_a(t)$

ideale - non realizzabile



banda di Nyquist

Spettro di Nyquist

$$\boxed{\begin{aligned} &D_{Ny}(f, \alpha) \\ &0 \leq \alpha \leq 1 \\ &B = W(1+\alpha) \\ &R_b = \frac{1}{T_b} = 2W \end{aligned}}$$

fattore di roll-off

velocità di Nyquist

bitrate

bitstream $\rightarrow$ [2-PAM] $\rightarrow$ [$g(t)$] $\rightarrow$ [$h(t)$] $\rightarrow \oplus \rightarrow$ [$c(t)$] $\rightarrow$ ⟋ $\rightarrow$ [DEC] $\rightarrow$ bitstream

$\sum_n a_n \delta(t - nT_b)$ $\quad\quad$ $w(t)$ $\quad\quad$ $t = nT_b$

$$y_s(mT_b) = \sum_n a_n\, d((m-m)T_b) = a_n\, d(0)$$

$$P_e = Q\left(\sqrt{SNR_c}\right) \qquad SNR_c = \frac{d_0^2 \cdot a^2}{P_{yw}} = \frac{d_0^2 a^2}{\int h_n |C(f)|^2 df}$$

$$= \frac{a^2 d_0^2}{h_n \int |C(f)|^2 df}$$

$\frac{N_0}{2}$

$$d(t) = g(t) * h(t) * c(t)$$

Se $h(t)$ passabasso B

$d(t) = g(t) * c(t) \cdot h_0$

$SNR_{c_{MAX}}$: $c(t) = g(-t)$ filtro adattato

$$d(t) = h_0 \cdot g(t) * c(t) = d_{Ny}(t, \alpha)$$ cond. suff.

$SNR_{c_{MAX}}$: $c(t) = g(-t)$
$C(f) = G(f)$
se $c(t)$ pari

$$\Longrightarrow \quad h_0 \, G(f) \, G(f) = D_{Ny}(f, \alpha)$$

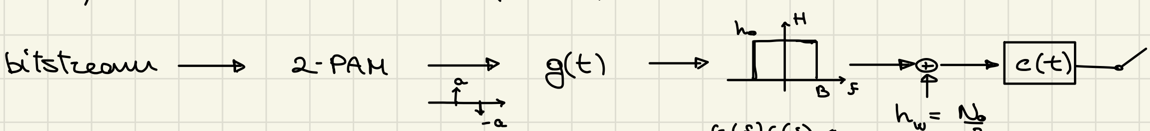
$$\boxed{G(f) = \sqrt{\frac{D_{Ny}(f,\alpha)}{h_0}}}$$

→ no ISI

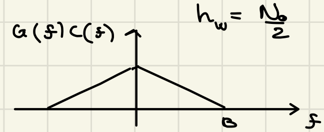
→ sí filtro adattato

<u>Es</u>: sensore acquisisce 100000 misure/s
ogni misura 16 bit
banda limitata B da determinare
attenuazione $L = 120\,dB$
rumore al ricevitore $\frac{N_0}{2} = 10^{-18}\ W/Hz$
ricostruisce al campionatore impulsi triang.

1) Bitrate + Schema (Tx - Rx)



$Rb = 100000 \times 16 = B$
$Rb = B = 1{,}6\ MHz$



2) Potenza media x bit

$$P_E = Q\left(\sqrt{SNR_c}\right) = \frac{1}{1{,}6 \cdot 10^6 \cdot 3600} = 1{,}7 \cdot 10^{-10}$$

$$SNR_c = \left(QINV\left(1{,}7 \cdot 10^{-10}\right)\right)^2 = 6{,}3^2$$

$$\frac{N_0}{2} = -180\,dB$$

$$y_s = a \cdot g(t) \cdot h_0 * g(-t) = a\, h_0\, r_g(t)$$
$$y_w = w(t) * g(-t)$$

$$L = \frac{1}{h_0}$$
$$120\,dB = 10 \log_{10}\left(\frac{1}{h_0^2}\right)$$
$$\frac{1}{h_0^2} = 240\,dB$$

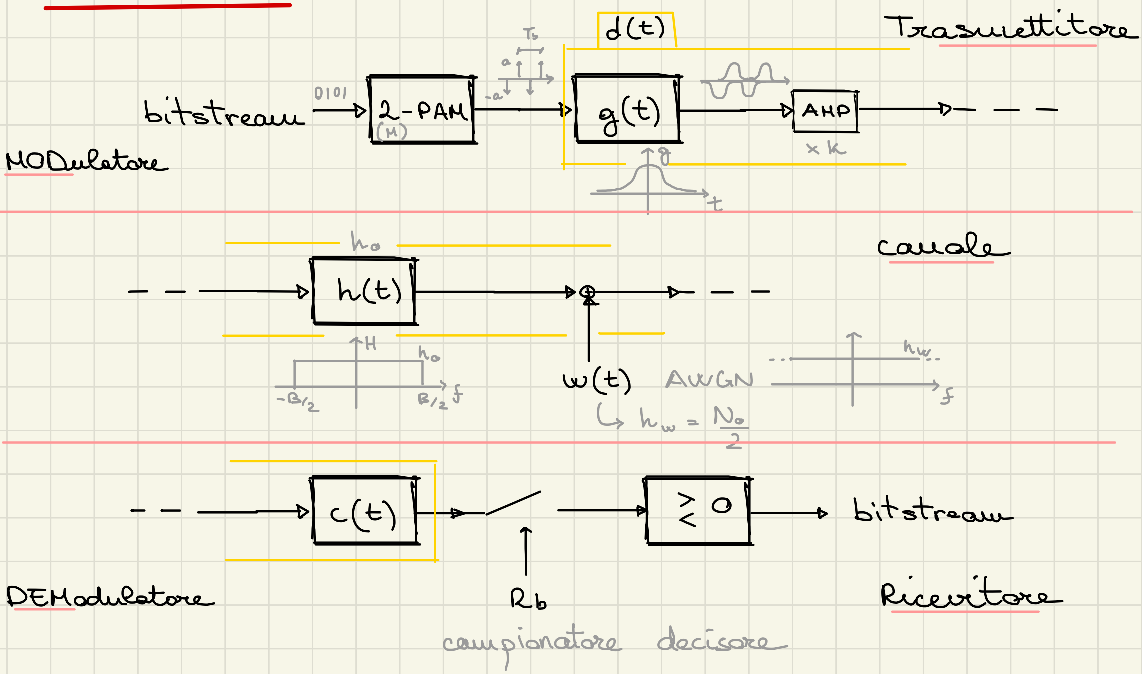
$$SNR_c = \frac{P_{y_s}}{P_{y_w}} = \frac{a^2 h_0^2 E_g^{\cancel{2}}}{\frac{N_0}{2}\cancel{E_g}} = 6{,}3^2 \longrightarrow E_g = \frac{6{,}3^2 \cdot \frac{N_0}{2}}{a^2 \cdot h_0^2}$$

P_{y_w}: $\int h_w |C(f)|^2 df$

filtro adattato

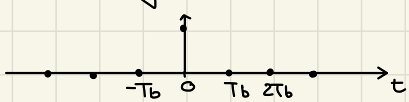
$$P_{T_b} = \frac{a^2 E_g}{T_b} = R_b \frac{1}{h_0^2} \frac{N_0}{2} \cdot (6{,}3)^2$$

Riassunto:



Inter Symbol Interference (ISI)

$\boxed{\text{NO ISI}} \rightarrow d(t) = g(t) * h(t) * c(t)$ SE $\boxed{d(t) = \begin{cases} d_0 & t = 0 \\ 0 & t \neq 0 \end{cases}}$



$$\|$$

$$\boxed{\sum_k D\left(f - \frac{k}{T_b}\right) = D_0}$$

$$D(f) = G(f)\,H(f)\,C(f)$$

$$\boxed{P_\varepsilon = Q\left(\sqrt{SNR_c}\right)} \quad SNR_c = \frac{(d(0)\cdot a)^2}{\int h_w |C(f)|^2 df} \overset{\text{F.A.}}{=} \frac{h_0^2 E_g^{\cancel{2}} a^2}{\frac{N_0}{2}\cdot \cancel{E_g}}$$

Filtro Adattato: $c(t) = g^*(-t)$ $\quad h(0) = h_0$

$$d(0) = h_0 \cdot g(t) * g^*(-t)\Big|_{t=0} \overset{\text{F.A}}{=} R_g(t)\Big|_{t=0} \cdot h_0 = E_g \cdot h_0$$

$$\boxed{SNR_c = \frac{h_0^2 a^2 E_g}{\frac{N_0}{2}} = \frac{2E_r}{N_0}}$$ → energia ricevuta

<u>Esercizi</u>:

- 5 immagini/min. $\quad$ 8 Mpix/imm. $\quad$ 24 bit/pix

  2 canali audio $\quad$ 44 kHz $\quad$ 24 bit/campione

  canale ideale $\quad$ attenuazione $L = 100\,dB$

  $N_0/2 = -200\,dB\ W/Hz$ $\quad$ mod. 2-PAM

1) bit rate ?

$$Rb|_{im} = 5\,\cancel{im}/\cancel{min} \cdot \frac{\cancel{min}}{60\,s} \cdot 8\cdot 10^6\,\cancel{pix}/\cancel{im} \cdot 24\,bit/\cancel{pix} = 16\cdot 10^6\,\frac{bit}{s}$$

$$Rb|_{aud} = 2 \cdot 44\cdot 10^3\,\cancel{camp}/s \cdot 24\,bit/\cancel{camp} = 2{,}112\cdot 10^6\,\frac{bit}{s}$$

$$Rb = 18{,}11\ Mbit/s$$

2) Potenza per bit tale da avere $p_e < 10^{-4}$

$$p_e = Q\left(\sqrt{SNR_c}\right) = 10^{-4} \Longrightarrow SNR_c = Qinv\left(10^{-4}\right)^2$$
$$= 13{,}8$$
$$= 11{,}4\ dB$$

$$SNR_c \overset{\text{F.A}}{=} \frac{E_r}{N_0/2} = \frac{P_r \cdot T_b}{N_0/2} = \frac{P_t \cdot h_0^2 \cdot T_b}{N_0/2} = 13{,}8$$

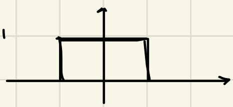
$1/h_0$ = attenuaz. = L

$$P_t = 13{,}8 \cdot \frac{N_0}{2} \cdot R_b \cdot \frac{1}{h_0^2} = 13{,}8 \cdot 10^{-20} \cdot 18{,}11 \cdot 10^5 \cdot 10^{10} = 24\,mW$$

3) Confrontare lo spettro dell'imp. trasmesso con quello di un sistema a imp. rect. a pari prestazioni

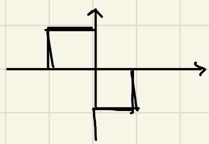
↳ stesso R_b, p_E

g_1



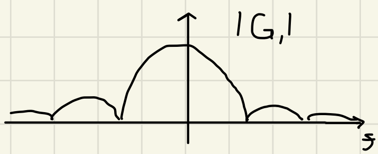
$\rightarrow \text{rect}\left(\frac{t}{2T}\right) \xrightarrow{\mathcal{F}} 2T\,\text{sinc}(2fT)$

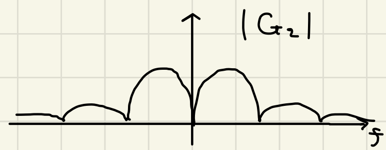
g_2



$\rightarrow \text{rect}\left(\frac{t+T/2}{T}\right) - \text{rect}\left(\frac{t-T/2}{T}\right)$

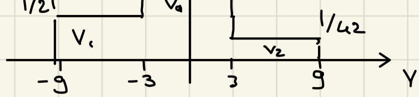
$\xrightarrow{\mathcal{F}} T\,\text{sinc}(fT)\,e^{+j\pi fT} - T\,\text{sinc}(fT)\,e^{-j\pi fT}$





- Trasm. segnale PRN da processo bianco con $B = 30\,kHz$ con prob. amp. quantizzato su 3 livelli

$f_c = 60\,kHz$

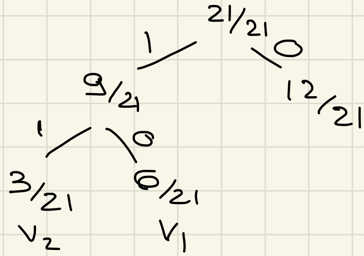


Utilizzare impulsi $\sqrt{Ny}$ $\alpha = 1$ roll-off
canale ideale passabasso B, $\frac{N_0}{2} = 10^{-20}\,\frac{W}{Hz}$ $L = 180\,dB$

$p_E = 1$ bit/ora

1) Codifica di Huffman, ridondanza media e entropia del bitrate

livello	prob.	codifica
V_0	$12/21$	0
V_1	$6/21$	10
V_2	$3/21$	11

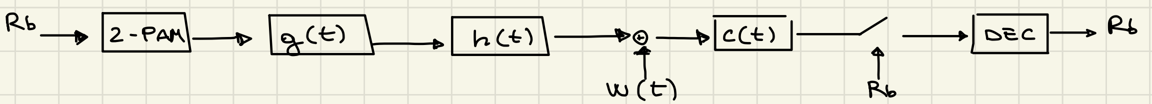


ridondanza media $= R = \sum_{i=1}^{3} l_i p_i = 1,43$ bit/camp.

$R_b = 1,43 \cdot 60\,kHz = 85,7$ kbit/s

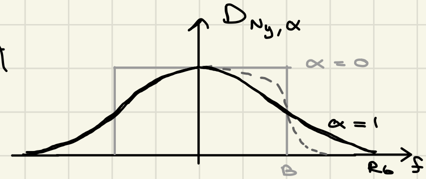
$H = -\sum_{i=1}^{3} p_i \log_2 p_i = 1,378$ bit/camp

2) Schema a blocchi e banda di $G(f)$, P_t per bit



$d(t) = g(t) * h(t) * c(t) = h_0\, g(t) * c(t)$

NO ISI $\quad \sum D\left(f - \frac{k}{T_b}\right) = \text{cost}$



$\alpha = 1 \longrightarrow B = R_b = 85,7$ KHz

$\underset{\uparrow \text{F.A.}}{D(f)} = h_0\, G^2(f) \longrightarrow G(f) = \sqrt{\frac{D_{Ny,\alpha}}{h_0}} =$

$= \sqrt{\frac{1}{h_0}\left(\frac{1}{2} + \frac{1}{2}\cos\left(\pi \frac{f}{B}\right)\right) \text{rect}\left(\frac{f}{2B}\right)}$

$P_E = Q\left(\sqrt{\frac{E_r}{N_0/2}}\right) = \frac{1 \text{ bit}}{\underbrace{85700 \cdot 3600}_{\text{bit in 1 ora}} \text{ bit}} = 3,2 \cdot 10^{-9}$

$$\frac{E_r}{N_0/2} = Q\text{inv}\left(3{,}2 \cdot 10^{-9}\right)^2 = 58^2$$

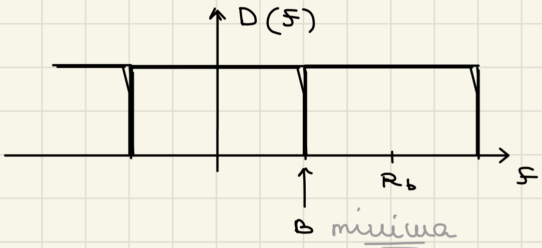
$$\frac{h_0^2 \cdot P_t \cdot T_b}{N_0/2} = 58^2$$

potenza trasmessa $\quad P_t = 58^2 \cdot \frac{1}{h_0^2} \cdot \frac{N_0}{2} \cdot R_b$

- impulsi antipodali $R_b = 15\ kb/s$ $\quad P_\varepsilon = 10^{-6}$

canale passabasso ideale

1) B minima necessaria e $g(t)$?



$$B = \frac{R_b}{2} = 7{,}5\ kHz$$

2) SNR_c ?

$$P_\varepsilon = Q\left(\sqrt{SNR_c}\right) = 10^{-6} \longrightarrow SNR_c = Q\text{inv}\left(10^{-6}\right)^2 = 22{,}6$$

3) Se aumento P_t di $+1dB$ quanti bit posso trasmettere in più in un'ora?

$$SNR_c = \frac{P_t/R_b \cdot h_0^2}{N_0/2} = 22{,}6 \qquad P_r$$

$$\frac{P_{t_2}}{R_{b_2}} = \frac{P_t}{R_b} \rightarrow R_{b_2} = 1{,}25\, R_b$$

$$P_{t_2} = 10^{0,1} P_t = 1{,}25\, P_t \longrightarrow \frac{P_{t_2}/R_{b_2} \cdot h_0^2}{N_0/2} = 22.6$$

4) Se aumento P_t di $+1\,dB$ quanto diminuisce la P_E?

$$\frac{1{,}25 \cdot P_t / R_b \cdot h_0^2}{N_0/2} = 22{,}6 \cdot 1{,}25$$

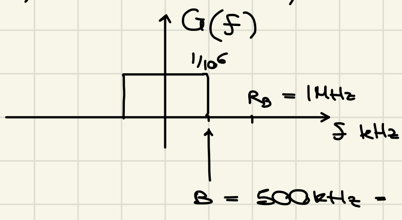
P_{t_2}

$$P_E = Q\left(\sqrt{22{,}6 \cdot 1{,}25}\right) = 4{,}8 \cdot 10^{-8}$$
$$= 2{,}5 \ \frac{bit}{ora}$$

- impulsi binari antipodali

$g(t) = \mathrm{sinc}(10^6 t) \qquad \frac{N_0}{2} = -200\,dB_{\frac{W}{Hz}} \qquad P_E = 10^{-8}$

1) F. adattato, R_b, B, P_t?



$$P_E = Q\left(\sqrt{SNR_c}\right) = 10^{-8} \rightarrow SNR_c = 31{,}4$$

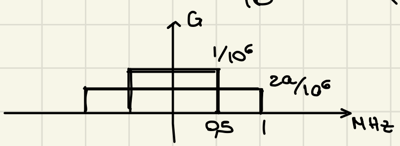
$$P_t = E_t R_b = SNR_c \cdot \frac{1}{h_0^2} \cdot \frac{N_0}{2} \cdot R_b$$
$$= 3{,}14 \ mW$$

2) $g(t) = \mathrm{sinc}(10^6 t) + a\, \mathrm{rect}(2 \cdot 10^6 t)$

a t.c. NO ISI? $\longrightarrow$ la sol. non è unica!

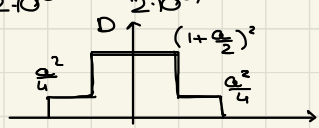
dipende da quale $f_c = R_b$ scelgo

$$G(f) = \frac{1}{10^6}\mathrm{rect}\left(\frac{f}{10^6}\right) + \frac{a}{2 \cdot 10^6}\mathrm{rect}\left(\frac{f}{2 \cdot 10^6}\right)$$



$\Longrightarrow$

F.A.



$\Downarrow$

$$\frac{a^2}{4} + \frac{a^2}{4} = \left(1 + \frac{a}{2}\right)^2$$

ripetizione nelle frequenze const. a $f = 1{,}5$ MHz

$$\Longleftarrow \quad \frac{a^2}{2} = 1 + \frac{a^2}{4} + a$$

$\Downarrow$

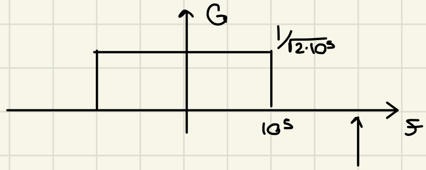
$$a = 2 \pm 2\sqrt{2}$$

- $$G(f) = \frac{1}{\sqrt{2\cdot 10^5}}\,\mathrm{rect}\left(\frac{f}{2\cdot 10^5}\right)$$

ampiezza $a = 1$, canale ideale banda B

$L = 80\,\mathrm{dB}$ $\quad \frac{N_0}{2} = -90\,\mathrm{dB}_{\frac{W}{Hz}}$

1) R_b ?



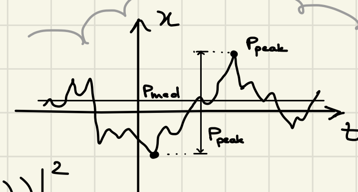
$R_b = 2\cdot 10^5\,\mathrm{Hz} = 200\,\mathrm{kHz}$

2) $P_E = ?$

$$P_E = Q\left(\sqrt{SNR_c}\right) = Q\left(\sqrt{\frac{E_r}{N_0/2}}\right) = Q\left(\sqrt{\frac{a^2 E_g\cdot h_c^2}{N_0/2}}\right)$$

$$E_g = \int |G(f)|^2 df = \int_{-10^5}^{10^5}\left|\frac{1}{\sqrt{2\cdot 10^5}}\right|^2 df = 1$$

$$\Longrightarrow P_E = Q\left(\sqrt{\frac{10^{-8}}{10^{-9}}}\right) = 7{,}8\cdot 10^{-3} \quad (\text{alta!})$$



$$P_{peak} = \max\left(|x(t)|^2\right)$$

$$P_{peak\text{-}peak} = \left|\max(x(t)) - \min(x(t))\right|^2$$

$$P_{med} = E\left[|x(t)|^2\right] = E[x(t)]^2 + \mathrm{var}[x(t)]$$

$$= \lim_{T_0\to\infty}\frac{1}{T_0}\int_{-T_0/2}^{T_0/2}|x(t)|^2\,dt$$ (segnali stazionari e ergodici)

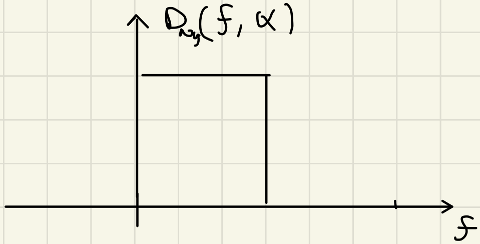
- $R_b = 750 \; k\dfrac{bit}{s}$ $\qquad B = 500 \; kHz$

1) α (roll-off) per occupare tutta la banda?

$$B = W(1+\alpha) = \frac{R_b}{2}(1+\alpha)$$

$$\alpha = \frac{2B}{R_b} - 1 = \frac{1000}{750} - 1 = 0{,}42$$

$0 \leq \alpha \leq 1$ sempre



2) È meglio α del p.to 1) o $\alpha = 0$?

Se mezzo ideale: $H(f) = h_0 \, rect\left(\frac{f}{2B}\right)$

F. A.: $\quad G(f) = C(f) = \sqrt{\dfrac{D_{Ny}(f,\alpha)}{h_0}}$

$$P_e = Q\left(\sqrt{SNR_c}\right) = Q\left(\sqrt{\frac{E_r}{N_0/2}}\right)$$

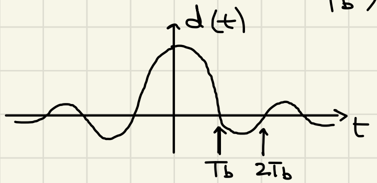
$$E_r = a^2 \int |G(f)|^2 df \; h_0^2$$

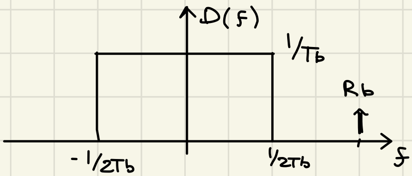
$$= a^2 \int D_{Ny}(f,\alpha) \, df \longrightarrow$$ NON dipende da α (integrale del coseno rialzato è simmetrico)

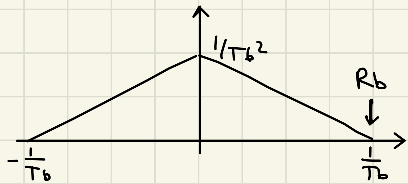
$\Longrightarrow$ α non influisce sulle prestazioni

- Campionatore ricostruisce impulsi di forma

① $d(t) = \mathrm{sinc}\left(\frac{t}{T_b}\right)$ ② $d(t) = \mathrm{sinc}^2\left(\frac{t}{T_b}\right)$







1) Velocità di trasmissione per non avere ISI?

$R_b = \frac{1}{T_b}$ dove i segnali si annullano per $t = \frac{n}{R_b}$ tranne in $n = 0$

sia per ①
che per ②

oppure

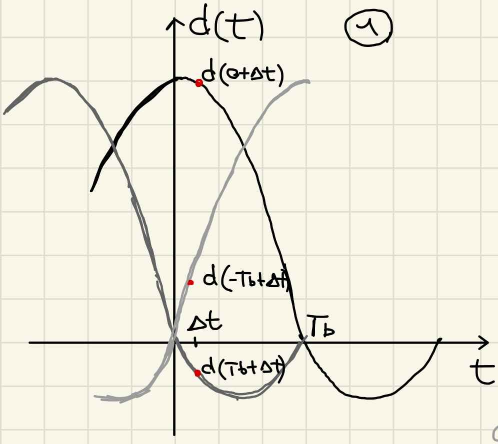
per avere una ripetizione della trasformata nelle frequenze const.

2) Banda occupata?

① $B = \frac{1}{2T_b}$ ② $B = \frac{1}{T_b}$ (unilatera)

3) Errore Δt al campionatore
$\Delta t \sim \mathcal{N}(0, \sigma_\Delta^2)$
Qual è la potenza del disturbo?
(rapporto segnale-errore)

1



segnale

$$y(0+\Delta t) = a_n d(0+\Delta t) + a_{n+1} d(T_b+\Delta t) + a_{n-1} d(-T_b+\Delta t)$$

rumore

$$P_s = E\left[|a_n d(0+\Delta t)|^2\right] = a^2 E\left[|d(\Delta t)|^2\right]$$

$$P_r = E\left[|a_{n+1} d(T_b+\Delta t) + a_{n-1} d(-T_b+\Delta t)|^2\right]$$

$$= E\left[|a_{n+1} x_1 + a_{n-1} x_{-1}|^2\right]$$

(considero solo le 2 interferenze più rilevanti)

$$= \overbrace{E[|a_{n+1}|]^2}^{a^2} E[|x_1|^2] + E[|a_{n-1}|^2] E[|x_{-1}|^2] + 2\,\underbrace{E[a_{n+1}]}_{0}\,\underbrace{E[a_{n-1}]}_{0}\,E[|x_1 x_2|]$$

$$= a^2 E[|x_1|^2] + a^2 E[|x_{-1}|^2]$$

$$\Longrightarrow \quad P_s = a^2 E[d(\Delta t)^2]$$

$$P_r = a^2 E[d(T_b+\Delta t)^2] + a^2 E[d(-T_b+\Delta t)]$$

$$d(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b}$$

$$d(0+\Delta t) \simeq 1$$

$$d(T_b+\Delta t) \simeq \frac{\Delta t/T_b}{\pi\left(1+\underbrace{\Delta t/T_b}_{\sim 0}\right)}$$

$$\Longrightarrow \quad P_s \simeq a^2$$

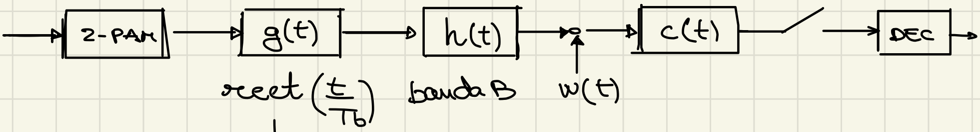
$$P_r \simeq a^2 \frac{\sigma_\Delta^2}{\pi^2 T_b^2} + a^2 \frac{\sigma_\Delta^2}{\pi T_b^2}$$

$$\frac{P_s}{P_r} = \frac{a^2}{a^2\, 2\, \frac{\sigma_\Delta^2}{\pi^2 T_b^2}} = \frac{\pi^2 T_b^2}{\sigma_\Delta^2}$$

- Trasmetto $\mathrm{rect}\left(\frac{t}{T_b}\right)$

mezzo con banda B (unilatera)

1) $C(f)$ tale che $D(f) = D_{Ny}(f, \alpha)$



$$G(f) = T_b \,\mathrm{sinc}(f T_b)$$

$$D(f) = G(f) H(f) C(f) = D_{Ny}(f, \alpha)$$

$$= T_b \,\mathrm{sinc}(f T_b) \cdot h_0 \,\mathrm{rect}\left(\frac{f}{2B}\right) C(f) = D_{Ny}(f, \alpha)$$

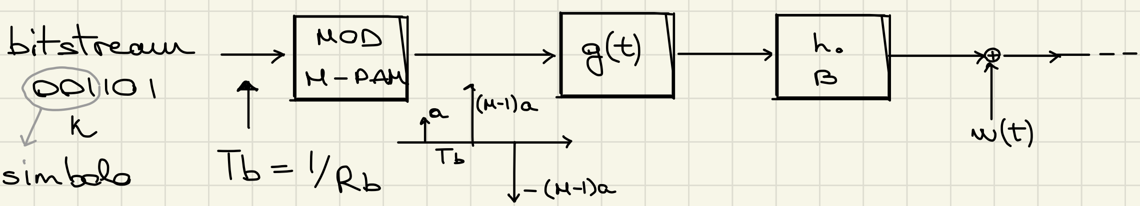
$$\longrightarrow \quad C(f) = \frac{D_{Ny}(f, \alpha)}{h_0 \,\mathrm{rect}\left(\frac{f}{2B}\right) T_b \,\mathrm{sinc}(f T_b)}$$

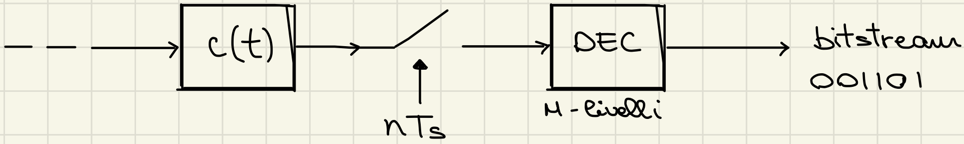
$$C(f) = \frac{D_{Ny}(f, \alpha)}{\mathrm{sinc}(f T_b)} \qquad |f| \leq B$$

Trasmissione multilivello

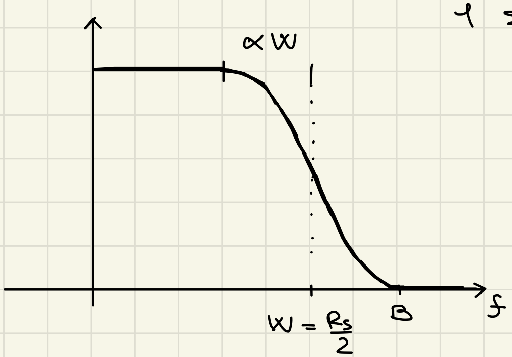
$$\left.\begin{array}{l} B \text{ limitata} + R_b \text{ elevato} \\ \quad B \leq 2R_b \end{array}\right\} \text{ molta potenza}$$

2 - PAM $\longrightarrow$ M - PAM





– Velocità di trasmissione



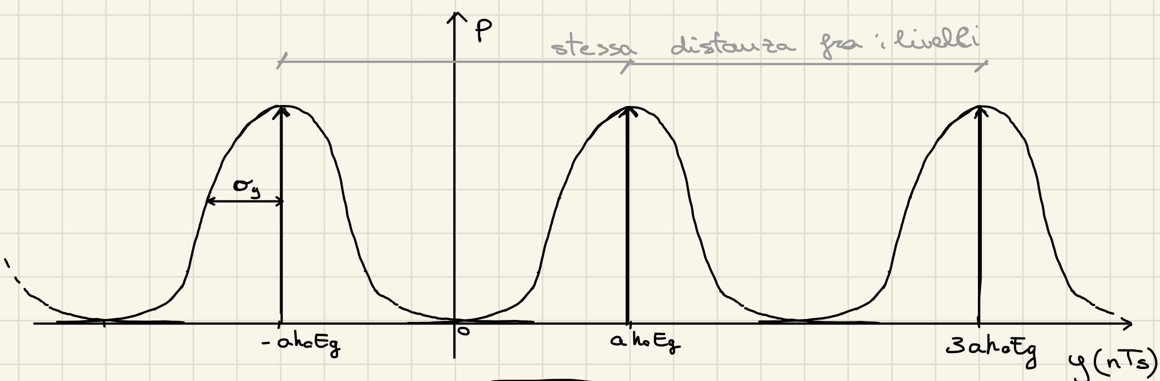
1 simbolo = k bit $\quad T_s = k T_b$

$$\boxed{R_s = \frac{R_b}{K}}$$

$$B = \frac{R_s}{2}(1+\alpha)$$

$$\boxed{B = \frac{R_b}{2k}(1+\alpha)}$$

– Probabilità di errore



$$\sigma_y = \sqrt{\frac{N_0}{2}\cdot E_g}$$

$$\boxed{P_{E_s} = \sum_{m=0}^{M-1} P_{E_s}\big|_m \cdot P(m) = \frac{1}{M}\sum_{m=0}^{M-1} P_{E_s}\big|_m = \frac{1}{M}\left(2(M-2)+2\right) Q\left(\frac{a h_0 E_g}{\sigma_y}\right) = \frac{2(M-1)}{M}\cdot Q\left(\sqrt{SNR_{c_s}}\right)}$$

$P_{E_b} = ?$ Hp: sbaglio x simboli adiacenti

codifica	Binaria	Gray
m_0	0 0 0	0 0 0
m_1	0 0 1	0 0 1
m_2	0 1 0	0 1 1

Gray > binaria

la distanza di Hamming fra due simboli consecutivi è sempre 1

$$\Longrightarrow \boxed{P_{E_b} = \frac{1}{K} \cdot P_{E_S} = \frac{2(M-1)}{KM} \cdot Q\left(\sqrt{SNR_{c_s}}\right)}$$

$$SNR_{c_s} = \frac{a^2 h_0^2 E_g^2}{\frac{N_0}{2} \cdot E_c} \overset{\text{F. A.}}{=} \frac{a^2 h_0^2 E_g}{N_0/2} = \frac{E_r}{N_0/2}$$

$(M-1)a$

$3a$

a

$-a$

$-3a$

$-(M-1)a$

livelli

$$= (M-1)^2 a^2 E_g$$

$$= 9a^2 E_g$$

$$\longrightarrow E = a^2 E_g$$

$$\boxed{\bar{E}_s = \frac{a^2 E_g}{M} + \frac{9a^2 E_g}{M} + \dots + \frac{(M-1)^2 a^2 E_g}{M} = \frac{1}{M} 2 \cdot a^2 \cdot E_g \left(1 + 3^2 + 5^2 + \dots + (M-1)^2\right) = \frac{M^2-1}{3} a^2 E_g}$$

$K = 1 \longrightarrow$ 2-PAM $\quad E = a^2 E_g$

$K = 2 \longrightarrow$ 4-PAM $\quad E = 5a^2 E_g$

$K = 3 \longrightarrow$ 8-PAM $\quad E = 21 a^2 E_g$

x2 velocità

x5 energia

$\bar{E}/\text{simb} \cdot \text{simb}/\text{bit}$

$$\left[\bar{P}_b = \frac{\bar{E}_b}{T_b} = \frac{\bar{E}_s \cdot 1/K}{T_b} = \frac{M^2-1}{3K}\, a^2 E_g R_b\right]$$

$$\longrightarrow \left[P_{E_b} = \frac{2(M-1)}{KM}\, Q\left(\sqrt{\frac{\bar{P}_b \cdot 3K}{M^2-1} \cdot \frac{h_0^2}{R_b} \cdot \frac{1}{N_0/2}}\right)\right]$$

Es: $f_c = 192$ KHz $\quad$ 24 bit/camp. $\quad$ M-PAM

$\frac{N_0}{2} = -200$ dB $\quad L = 70$ dB $\quad B = 1$ MHz

$P_E = 10^{-12}$

$\bar{P}_b = ?$ $\quad$ Potenza media x bit

$R_b = 192000 \times 24 = 46 \frac{\text{Mbit}}{\text{s}}$

$(0 \le \alpha \le 1)$

$$\frac{R_s}{2}(1+\alpha) \le 10^6 \longrightarrow \frac{R_b}{2K}(1+\alpha) \le 10^6 \longrightarrow \frac{23}{K}(1+\alpha) \le 1$$

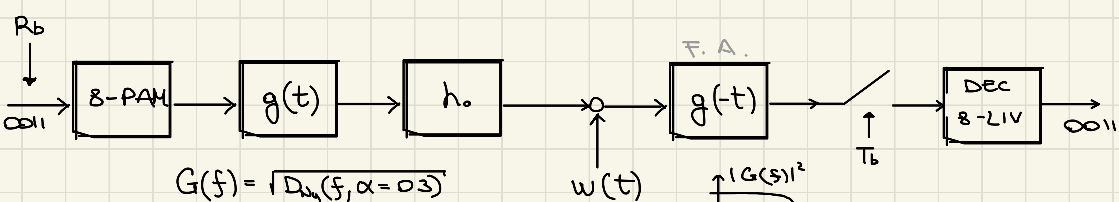
$$\Downarrow$$

$$\frac{23}{3}(1+\alpha) \le 1 \longleftarrow K = 3$$

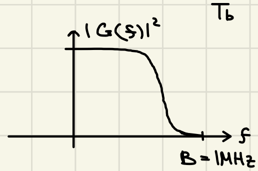
$$\Downarrow \qquad\qquad \Downarrow$$

$$M = 8$$

$$\alpha = \frac{3}{23} - 1 = 0,3$$



$G(f) = \sqrt{D_{Ny}(f, \alpha = 0{,}3)}$



M-PAM $\quad P_E = Q\left(\sqrt{SNR_c}\right)\cdot \frac{2(M-1)}{M\cdot K} = 10^{-12} \longrightarrow SNR_c = Qinv^2\left(10^{-12}\cdot \frac{12}{7}\right)$

$$= 16{,}8 \text{ dB}$$

P_{tb}/R_b

$$SNR_c = \frac{E_b \cdot 3K}{M^2-1}\cdot \frac{h_0^2}{N_0/2} =|_{dB}\ 16{,}8\,dB$$

$$P_{tb} = \underset{SNR}{16{,}8\text{ dB}} - \underset{N_0/2}{200\text{dB}} + \underset{L}{70\text{dB}} + \underset{R_b}{66{,}72\text{dB}} + \underset{\frac{M^2-1}{3K}}{8{,}48\text{dB}} = -38{,}1\text{dB}$$

2-PAM $\quad P_E = Q\left(\sqrt{SNR_c}\right) = 10^{-12} \longrightarrow SNR_c = Qinv^2(10^{-12})$

$$= 16{,}9\text{dB}$$

$$SNR_c = \frac{E_b \cdot h_0^2}{N_0/2} =|_{dB}\ 16{,}9\text{dB}$$

$$P_{tb} = 16{,}9\text{dB} - 200\text{dB} + 70\text{dB} + 66{,}72\text{dB} = -46{,}58\text{dB}$$

K	M	$E_{peak} = (M-1)^2$	$\bar{E}_b = \frac{M^2-1}{3K}$
1	2	1	1
2	4	9	15/6
3	8	36	7
4	16	225	85/4

$\longrightarrow$ di solito non si va oltre il 64-PAM

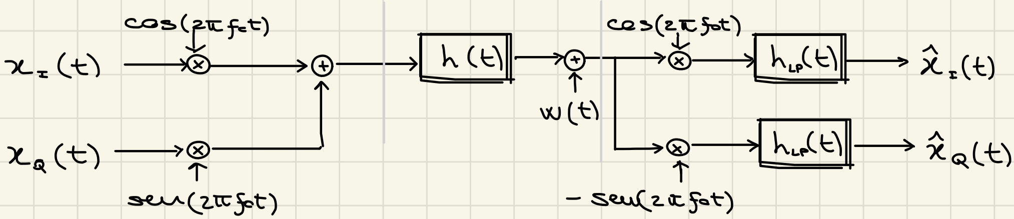
Trasmissione banda traslata

Serve per trasmettere su lunghe distanze (ad alta frequenza)

$$x(t) \longleftrightarrow X(f)$$

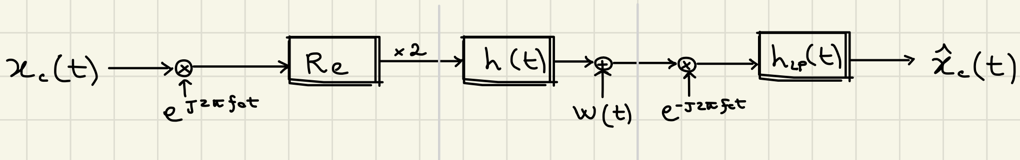
$$x(t)\cos(2\pi f_0 t) \longleftrightarrow \frac{1}{2}X(f-f_0) + \frac{1}{2}X(f+f_0)$$

$$y(t)\,\text{sen}(2\pi f_0 t) \longleftrightarrow$$

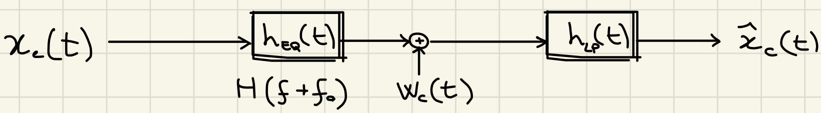


$$x_I(t)\ \cos(2\pi f_0 t)\cdot h_0\cdot \cos(2\pi f_0 t) = \frac{h_0 x_I(t)}{2} + \frac{h_0 x_I(t)}{2}\ \cancel{\cos(2\pi 2 f_0 t)}^{\,LP}$$

$$x_Q(t)\cdot sen(2\pi f_0 t)\cdot h_0\cdot \cos(2\pi f_0 t) = x_Q(t)\, 2 h_0\, \cancel{sen}^{\,LP}(2\pi 2 f_0 t)$$



$$x_c(t) = x_I(t) + j\, x_Q(t)$$



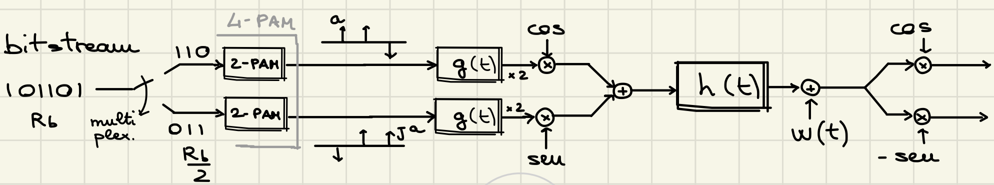
modello equivalente in banda base

$$\underbrace{W_c(t)}_{N_0 B} = w(t)\cdot e^{-j2\pi f_0 t} = \underbrace{w_I(t)}_{\frac{N_0 B}{2}}\cos(2\pi f_0 t) - \underbrace{w_Q(t)}_{\frac{N_0 B}{2}} sen(2\pi f_0 t)$$

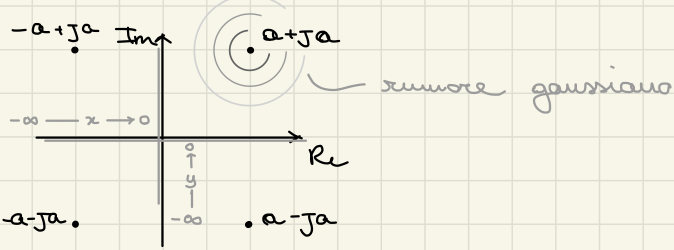
$$P_w = 2\frac{N_0}{2}\cdot B = N_0 B$$

$$E[w_I(t)\, w_Q^*(t)] = 0$$

$$\sigma^2_{w_I} = \frac{1}{T_0}\int E[w_I^2(t)\cos^2(2\pi f_0 t)]\,dt = \frac{N_0 B}{2}$$



integro per trovare P_e



$$y(t)\Big|_{t=nT_b} = a_k d(0) + w_c(t) * c(t)$$

$$w_c = w_t + w_a \qquad \frac{N_0}{2} \cdot 2B \cdot E_c$$

$$\mathcal{N}(0, \sigma_{w_c}^2) \qquad \mathcal{N}(0, \sigma_{w_c}^2)$$

di poco minore

$$P_{e_s} \simeq \int_{-\infty}^{0}\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-a)^2}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y-a)^2}{2\sigma^2}} dx\, dy$$

$$= 2Q\left(\frac{a\, d(0)\, h_0}{N_0 B E_c}\right)$$

$$P_{e_b} = Q\Big($$

$$x_I$$

$$x_Q$$

$$y(t)\Big|_{t=0} = a_k\, g(t) * g(-t)\, h_0 = a_k E_g h_0$$

$\pm a \pm ja$

$$w(t) \rightarrow P_w = \frac{N_0}{2} E_g \qquad SNR_c = \frac{E[|a_k E_g h_0|^2]}{\frac{N_0}{2} E_g}$$

$$= \frac{2a^2 E_{\cancel{g}}^{\cancel{2}} h_0^2}{\frac{N_0}{2} \cancel{E_g}} = \frac{2a^2 E_g h_0^2}{\frac{N_0}{2}}$$